

t_{ij} の概念とそれに基づく正規圧密土 のモデル化

tij地盤解析研究所
(株)地域 地盤 環境 研究所
中部大学
元名古屋工業大学

中井照夫



1

内容

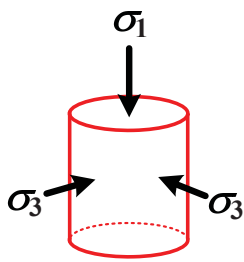
1. 弾塑性論の基本
 - 1次元の弾性および弾塑性論
 - 多次元の弾性および弾塑性論
2. 地盤材料の1次元挙動とモデル化
 - 正規圧密土
 - 過圧密土
 - 自然堆積土
 - 正規圧密土から自然堆積土までの時間効果特性
3. 地盤材料の通常の多次元モデル
 - 正規圧密土から過圧密土までのせん断挙動
 - 弾・完全塑性モデル (Drucker-Prager model, Mohr-Coulomb model)
 - Cam clay model
4. t_{ij} の概念とそれに基づく正規圧密土のモデル化 (t_{ij} clay model)
 - t_{ij} の概念に基づく正規圧密土のモデル化
 - 滑動面から t_{ij} へ (付録)
 - t_{ij} の概念の意味
5. t_{ij} の概念に基づく過圧密土から自然堆積土までのモデル化 (Subloading t_{ij} model)
 - 過圧密土
 - 自然堆積土
 - 正規圧密土から自然堆積土までの時間効果特性
6. 陰解法によるモデルの定式化
 - 引張り応力のでない弾性式
 - Return Mapping
7. 繰返し载荷特性のモデル化他

2

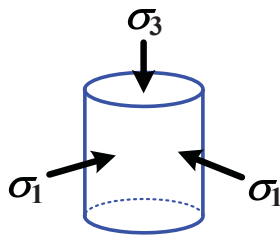
t_{ij} の概念に基づく正規圧密土のモデル化 (tij clay model)

3

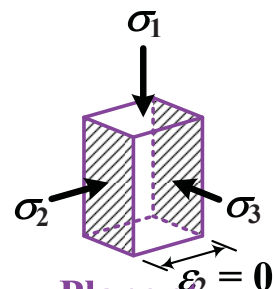
要素試験の応力状態



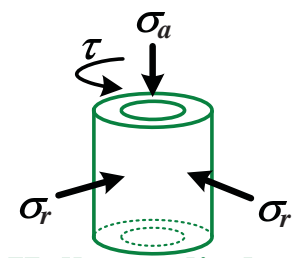
Comp.
三軸圧縮



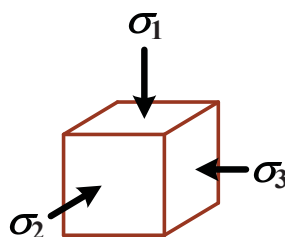
Ext.
三軸伸張



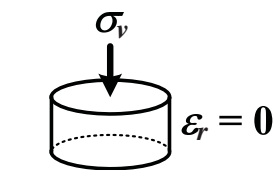
Plane st.
平面ひずみ



Hollow cylinder
中空ねじり



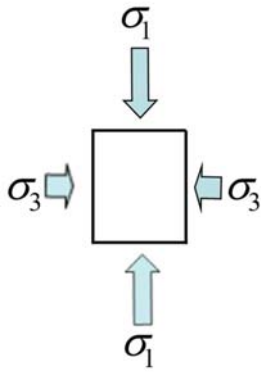
True triaxial
真三軸



Oedometer
標準圧密

4

2次元状態での応力の不変量

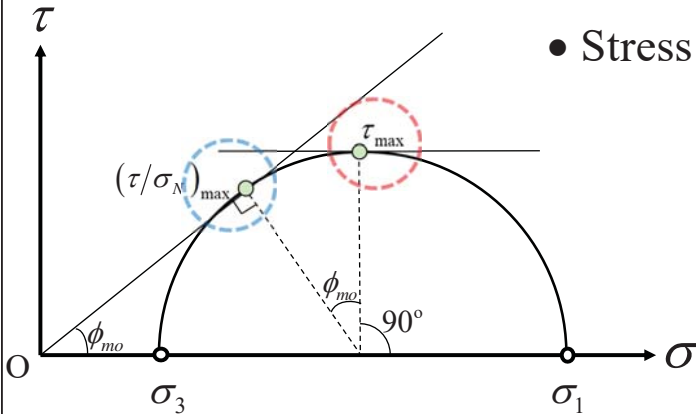


- Stress and stress ratio on τ_{\max} plane:

$$\sigma_{45^\circ} = s = \frac{\sigma_1 + \sigma_3}{2}$$

$$\tau_{45^\circ} = t = \frac{\sigma_1 - \sigma_3}{2}$$

$$\frac{\tau_{45^\circ}}{\sigma_{45^\circ}} = \frac{t}{s} = \sin \phi_{mo} = \frac{\sigma_1 - \sigma_3}{\sigma_1 + \sigma_3}$$



- Stress and stress ratio on $(\tau / \sigma_N)_{\max}$ plane:

$$\sigma_N = \frac{2\sigma_1\sigma_3}{\sigma_1 + \sigma_3}$$

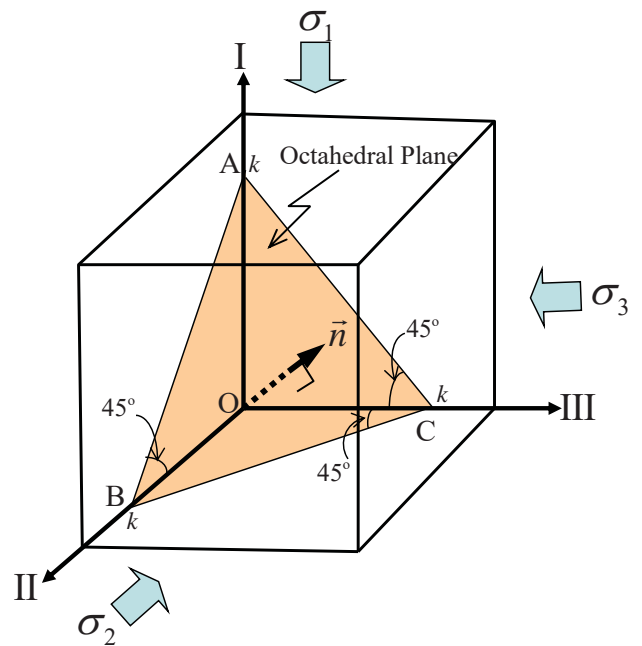
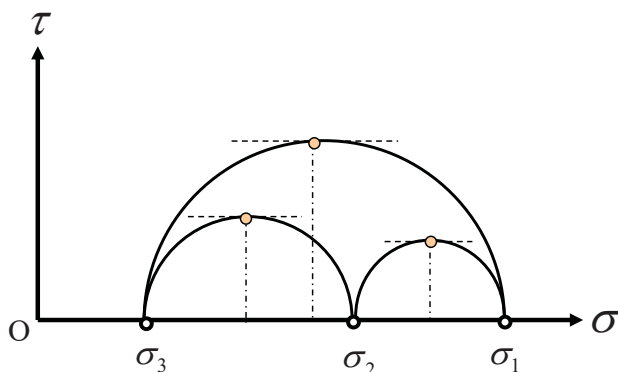
$$\tau = \frac{(\sigma_1 - \sigma_3)\sqrt{\sigma_1\sigma_3}}{\sigma_1 + \sigma_3}$$

$$\frac{\tau}{\sigma_N} = \tan \phi_{mo} = \frac{\sigma_1 - \sigma_3}{2\sqrt{\sigma_1\sigma_3}} = \frac{1}{2} \left(\sqrt{\frac{\sigma_1}{\sigma_3}} - \sqrt{\frac{\sigma_3}{\sigma_1}} \right)$$

村山(1964)

5

3次元応力状態と正八面体面



正八面体面の法線方向の単位ベクトル

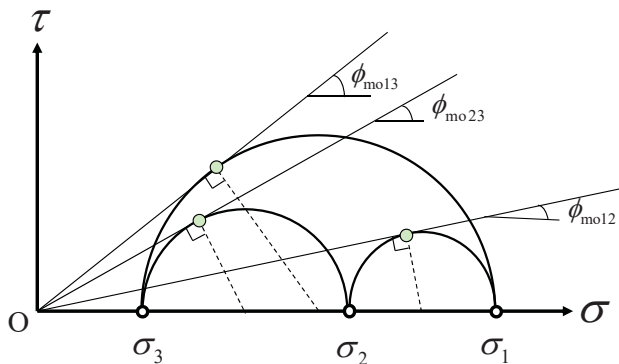
$$\frac{\vec{n}}{|\vec{n}|} = \left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}} \right)$$

6

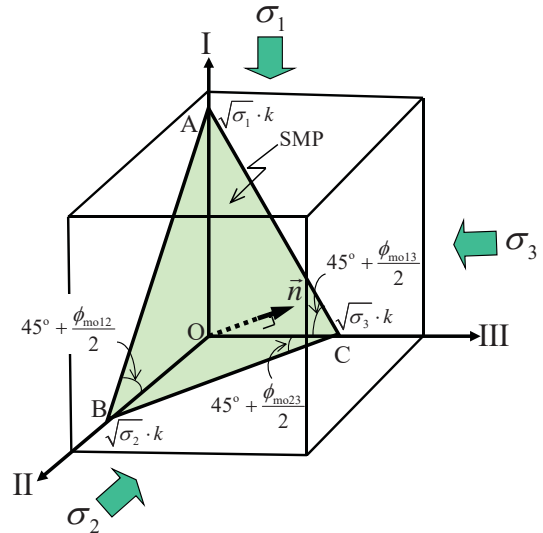
3次元応力状態と空間活動面

Spatially mobilized plane (SMP)

松岡・中井(1974)



$$\tan\left(45^\circ + \frac{\phi_{moij}}{2}\right) = \sqrt{\frac{1 + \sin \phi_{moij}}{1 - \sin \phi_{moij}}} = \sqrt{\frac{\sigma_i}{\sigma_j}} \quad (i, j=1, 2, 3; i < j)$$



SMPの法線方向の方向余弦

$$\frac{\vec{n}}{|\vec{n}|} = (a_1, a_2, a_3)$$

$$a_1 = \sqrt{\frac{I_3}{I_2\sigma_1}}, \quad a_2 = \sqrt{\frac{I_3}{I_2\sigma_2}}, \quad a_3 = \sqrt{\frac{I_3}{I_2\sigma_3}}$$

(I_2, I_3 : 応力の2次、3次の不変量)

$$\left. \begin{aligned} I_1 &= \sigma_1 + \sigma_2 + \sigma_3 = \sigma_{ii} \\ I_2 &= \sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_3\sigma_1 = \frac{1}{2}\left\{(\sigma_{ii})^2 - \sigma_{ij}\sigma_{ji}\right\} \\ I_3 &= \sigma_1\sigma_2\sigma_3 = \frac{1}{6}e_{ijk}e_{lmn}\sigma_{il}\sigma_{jm}\sigma_{kn} \end{aligned} \right\}$$

修正応力 t_{ij}

中井・三原(1984)

- $t_1 = a_1\sigma_1, \quad t_2 = a_2\sigma_2, \quad t_3 = a_3\sigma_3$

(a_1, a_2 and a_3) はSMPの法線方向の単位ベクトル(方向余弦)

$$a_1 = \sqrt{\frac{I_3}{I_2\sigma_1}}, \quad a_2 = \sqrt{\frac{I_3}{I_2\sigma_2}}, \quad a_3 = \sqrt{\frac{I_3}{I_2\sigma_3}}$$

- $t_{ij} = a_{ik}\sigma_{kj}$

a_{ij} は(a_1, a_2 and a_3)を主値とする対称テンソルなので、 t_{ij} も σ_{ij} と共軸な対称テンソル

$$a_{ij} = \sqrt{\frac{I_3}{I_2}} \cdot r_{ij}^{-1} = \sqrt{\frac{I_3}{I_2}} \cdot (\sigma_{ik} + I_{r2}\delta_{ik})(I_{r1}\sigma_{kj} + I_{r3}\delta_{kj})^{-1}$$

$$\left(\begin{array}{l} \text{where } r_{ik}r_{kj} = \sigma_{ij} \\ I_{r1}, I_{r2}, I_{r3} : \text{1st, 2nd, 3rd stress invariants of } r_{ij} \end{array} \right)$$

$$a_{ij} = Q_{im}Q_{jn}\hat{a}_{mn}$$

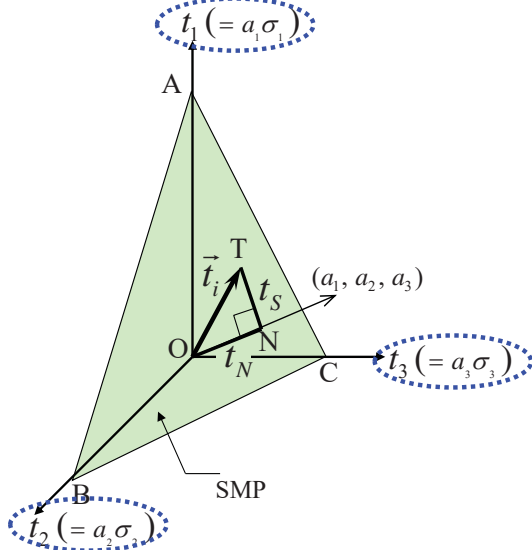
$$\left(\begin{array}{l} \text{where } \hat{a}_{mn} = \begin{pmatrix} a_1 & 0 & 0 \\ 0 & a_2 & 0 \\ 0 & 0 & a_3 \end{pmatrix}, \quad Q_{ij} : \text{transformation matrix defined by eigenvector of } \sigma_{ij} \end{array} \right)$$

t_{ij} に基づく3次元モデルの定式化(1)

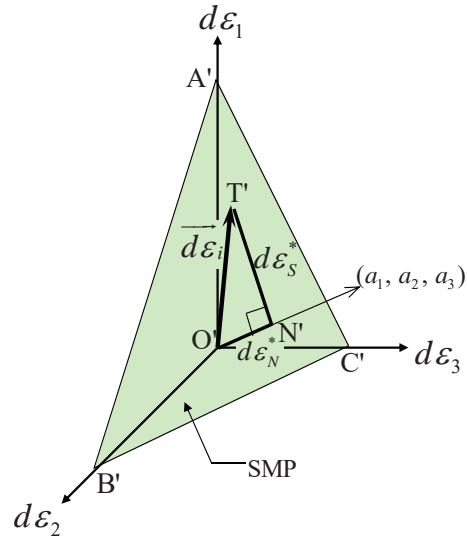
中井・三原(1984)

修正応力 $t_1 = a_1\sigma_1, t_2 = a_2\sigma_2, t_3 = a_3\sigma_3$ を用いて

応力パラメーター:



ひずみ増分パラメーター



$$\begin{cases} t_N = t_1 a_1 + t_2 a_2 + t_3 a_3 \\ t_S = \sqrt{(t_1 a_2 - t_2 a_1)^2 + (t_2 a_3 - t_3 a_2)^2 + (t_3 a_1 - t_1 a_3)^2} \end{cases}$$

$$\begin{cases} d\varepsilon_N^* = d\varepsilon_1 a_1 + d\varepsilon_2 a_2 + d\varepsilon_3 a_3 \\ d\varepsilon_S^* = \sqrt{(d\varepsilon_1 a_2 - d\varepsilon_2 a_1)^2 + (d\varepsilon_2 a_3 - d\varepsilon_3 a_2)^2 + (d\varepsilon_3 a_1 - d\varepsilon_1 a_3)^2} \end{cases}$$

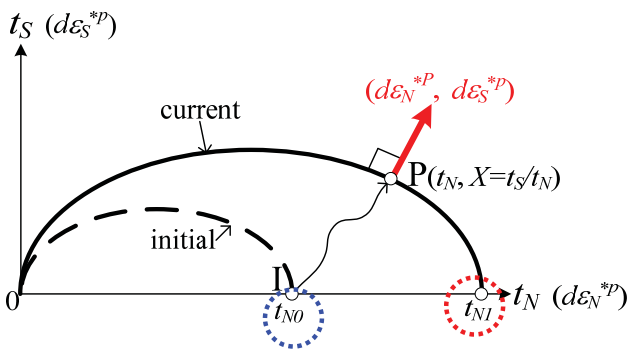
t_{ij} に基づく3次元モデルの定式化(2)

正規圧密土の1次元モデルを3次元化 (Cam clay modelに t_{ij} の概念を導入)

• 降伏関数 (塑性ポテンシャル)

$$F = H \text{ or } f = F - H = 0$$

$$\begin{cases} F = \ln \frac{t_{N1}}{t_{N0}} = \ln \frac{t_N}{t_{N0}} + \zeta(X) \\ H = \frac{(-\Delta e)^p}{\lambda - \kappa} = \frac{1 + e_0}{\lambda - \kappa} \varepsilon_v^p \end{cases} \quad (\text{where } X = t_S / t_N)$$



↓ ← 適応条件: $df = 0$

• 流れ則

$$d\varepsilon_{ij}^p = \Lambda \frac{\partial F}{\partial t_{ij}}$$

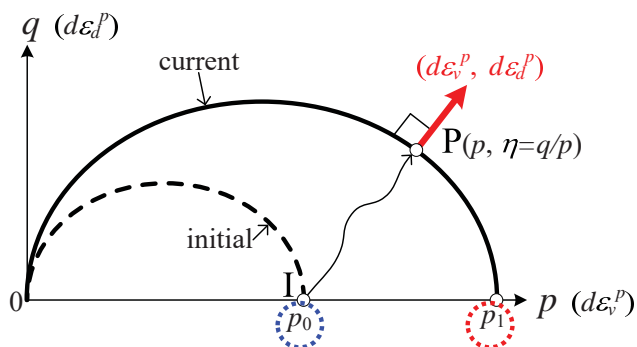
$$\Lambda = \frac{dF}{\frac{1+e_0}{\lambda-\kappa} \frac{\partial F}{\partial t_{kk}}} = \frac{dF}{h^p} \quad (\text{where } dF = \frac{\partial F}{\partial \sigma_{ij}} d\sigma_{ij})$$

$$\begin{aligned} df &= dF - dH = dF - \frac{\partial H}{\partial \varepsilon_{ij}^p} d\varepsilon_{ij}^p \\ &= dF - \frac{\partial H}{\partial \varepsilon_{ij}^p} \frac{\partial F}{\partial t_{ij}} \Lambda \\ &= dF - \frac{1+e_0}{\lambda-\kappa} \delta_{ij} \frac{\partial F}{\partial t_{ij}} \Lambda \\ &= dF - \frac{1+e_0}{\lambda-\kappa} \frac{\partial F}{\partial t_{ii}} \Lambda = 0 \end{aligned}$$

参考: Cam clay modelの定式化

•降伏関数(塑性ポテンシャル)

$$F = H \text{ or } f = F - H = 0$$



$$\begin{cases} F = \ln \frac{p_1}{p_0} = \ln \frac{p}{p_0} + \zeta(\eta) \\ H = \frac{(-\Delta e)^p}{\lambda - \kappa} = \frac{1 + e_0}{\lambda - \kappa} \varepsilon_v^p \end{cases}$$

↓ ← 適応条件: $df = 0$

•流れ則

$$d\varepsilon_{ij}^p = \Lambda \frac{\partial F}{\partial \sigma_{ij}}$$

$$\Lambda = \frac{dF}{\frac{1+e_0}{\lambda-\kappa} \frac{\partial F}{\partial \sigma_{kk}}} = \frac{dF}{h^p} \quad (\text{where } dF = \frac{\partial F}{\partial \sigma_{ij}} d\sigma_{ij})$$

$$\begin{aligned} df &= dF - dH = dF - \frac{\partial H}{\partial \varepsilon_{ij}^p} d\varepsilon_{ij}^p \\ &= dF - \frac{\partial H}{\partial \varepsilon_{ij}^p} \frac{\partial F}{\partial \sigma_{ij}} \Lambda \\ &= dF - \frac{1+e_0}{\lambda-\kappa} \delta_{ij} \frac{\partial F}{\partial \sigma_{ij}} \Lambda \\ &= dF - \frac{1+e_0}{\lambda-\kappa} \frac{\partial F}{\partial \sigma_{ii}} \Lambda = 0 \end{aligned}$$

11

降伏関数の形を決める関数 $\zeta(X)$ について(1)

•降伏関数(塑性ポテンシャル)

$$F = \ln \frac{t_{N1}}{t_{N0}} = \ln \frac{t_N}{t_{N0}} + \zeta(X)$$

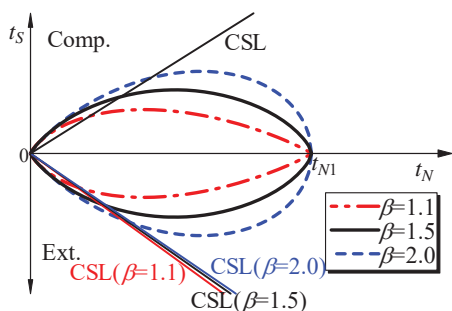
$$\zeta(X) = \frac{1}{\beta} \left(\frac{X}{M^*} \right)^\beta$$

($X = 0$ を満たす増加関数)

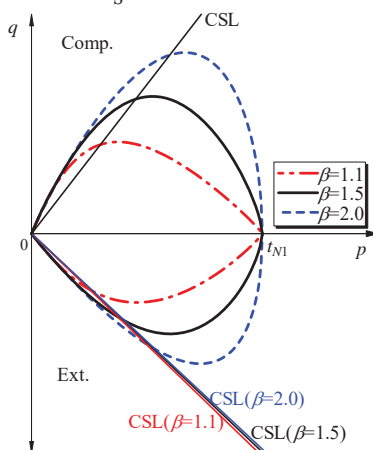
Chowdhury & Nakai (1998)

$$\frac{d\varepsilon_N^*}{d\varepsilon_S^*} = \frac{\frac{\partial F}{\partial t_N}}{\frac{\partial F}{\partial t_S}} = \frac{\frac{\partial F}{\partial t_N} + \frac{\partial F}{\partial X} \frac{\partial X}{\partial t_N}}{\frac{\partial F}{\partial X} \frac{\partial X}{\partial t_S}} = \frac{1 - \zeta'(X) \cdot X}{\zeta'(X)} = \frac{(M^*)^\beta - X^\beta}{X^{\beta-1}}$$

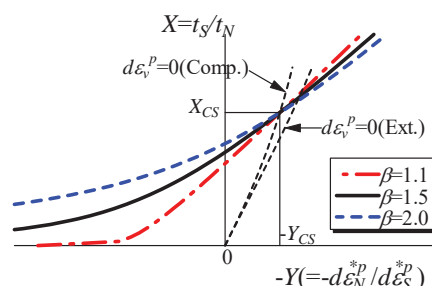
$$M^* = (X_{CS}^\beta + X_{CS}^{\beta-1} Y_{CS})^{1/\beta}$$



t_S - t_N 平面での降伏曲面



p - q 平面での降伏曲面

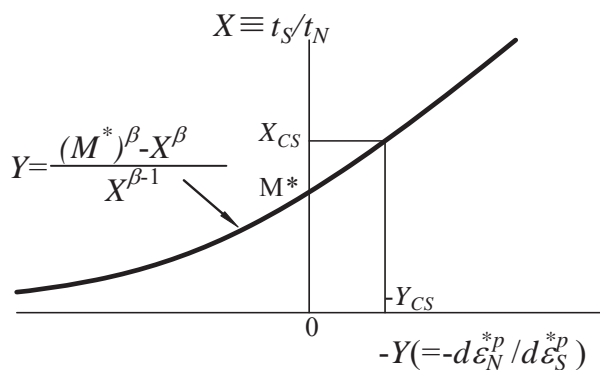


Stress-dilatancy関係

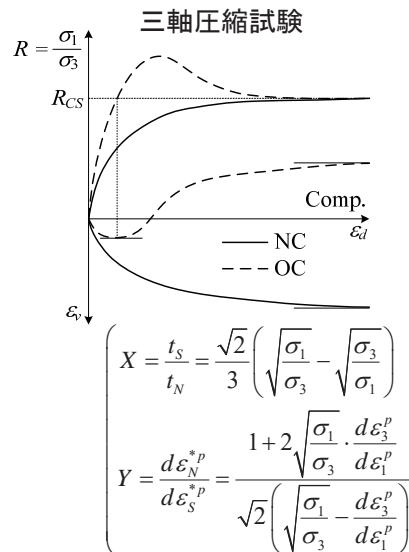
12

降伏関数の形を決める関数 $\zeta(X)$ について(2)

●降伏関数の形の決め方



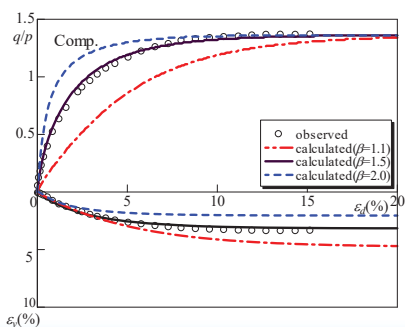
$$M^* = \left(X_{CS}^\beta + X_{CS}^{\beta-1} Y_{CS} \right)^{1/\beta}$$



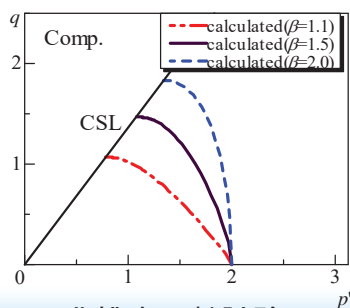
↓

限界状態(critical state)では

$$\begin{cases} X_{CS} = \frac{\sqrt{2}}{3} \left(\sqrt{R_{CS}} - \frac{1}{\sqrt{R_{CS}}} \right) \\ Y_{CS} = \frac{1 - \sqrt{R_{CS}}}{\sqrt{2} (\sqrt{R_{CS}} + 0.5)} \end{cases}$$

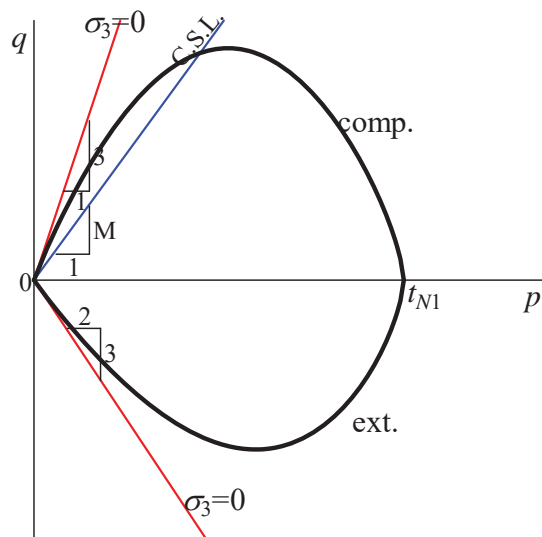
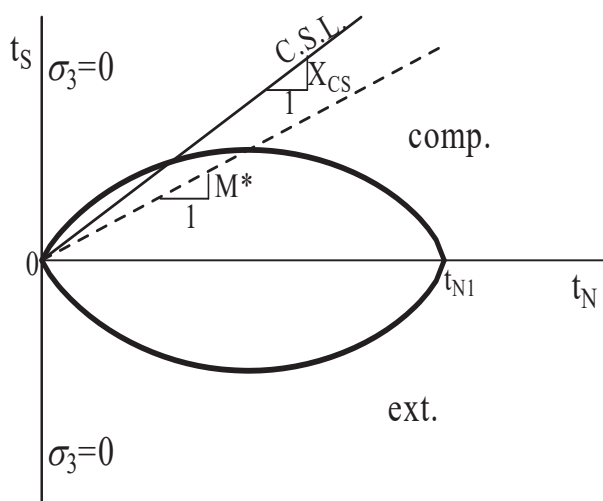


排水三軸試験

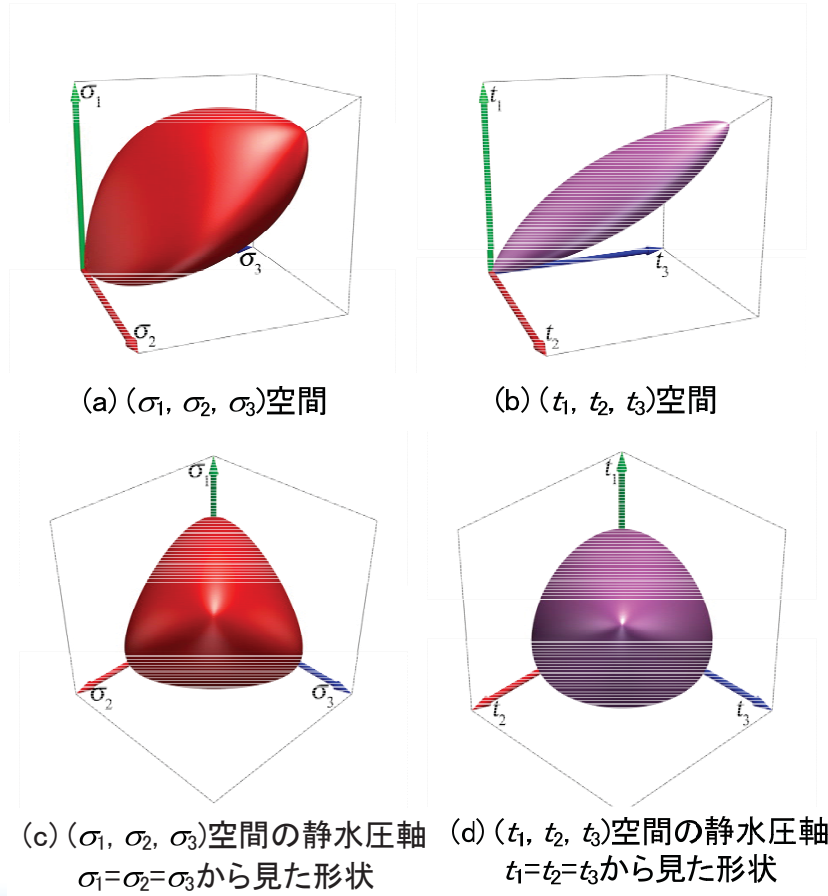


非排水三軸試験

t_{ij} modelの降伏曲面(2次元平面での表示)

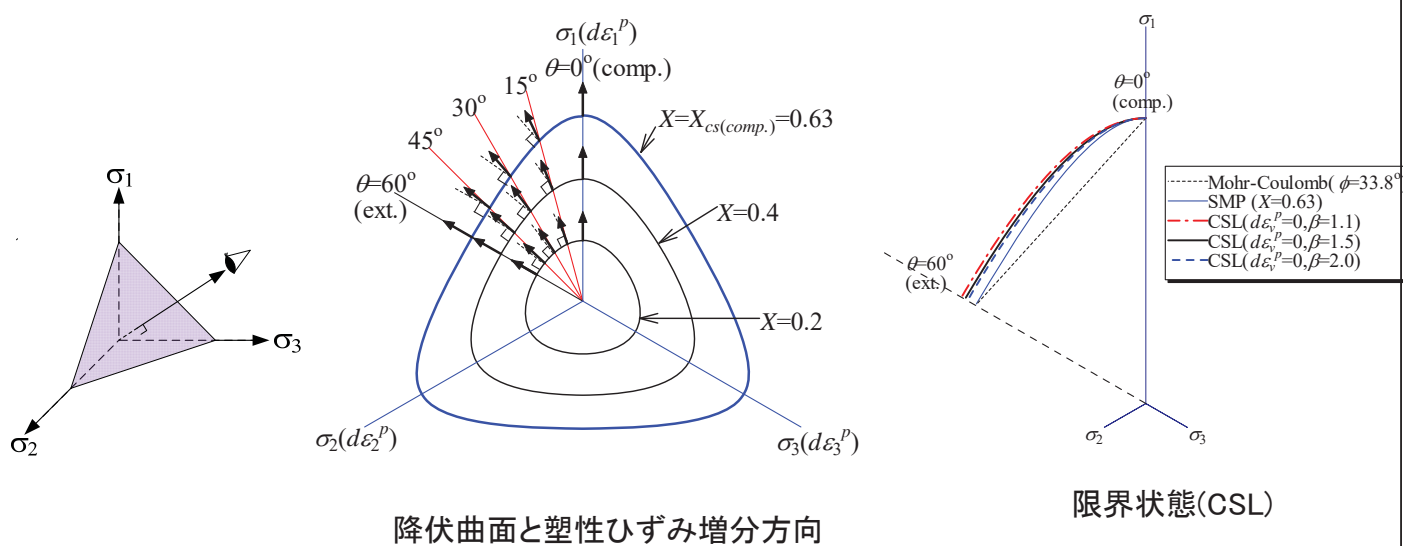


t_{ij} modelの降伏曲面(3次元空間での表示)



15

$(\sigma_1, \sigma_2, \sigma_3)$ 空間の正八面体面上での t_{ij} modelの考察-



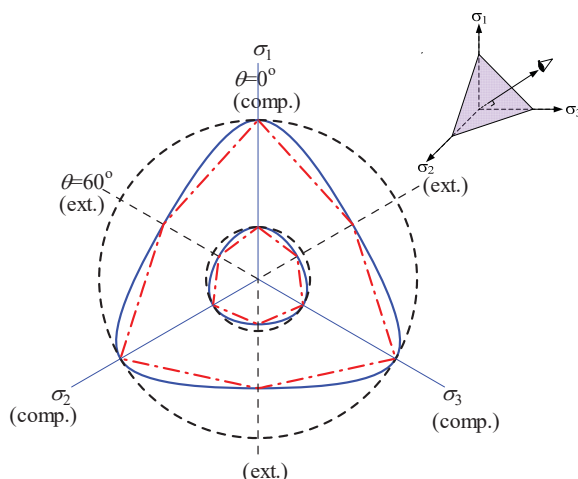
- σ_{ij} 空間ではなく t_{ij} 空間で流れ則を考えるので、 $\sigma_1 > \sigma_2 > \sigma_3$ の相異なる3主応力下では塑性ひずみ増分は半径方向と降伏曲面に垂直方向の間の方向になる
- 伸張側の限界状態はモール・クーロン基準やSMP基準のそれよりも大きくなる

16

SMP規準

松岡・中井(1974)

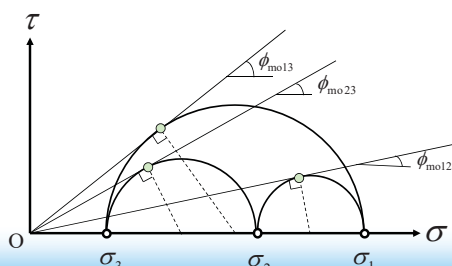
$$\begin{aligned}
 t_N &= t_1 a_1 + t_2 a_2 + t_3 a_3 \\
 &= \sigma_1 a_1^2 + \sigma_2 a_2^2 + \sigma_3 a_3^2 = 3 \frac{I_3}{I_2} = \sigma_{SMP} \\
 t_S &= \sqrt{(t_1 a_2 - t_2 a_1)^2 + (t_2 a_3 - t_3 a_2)^2 + (t_3 a_1 - t_1 a_3)^2} \\
 &= \sqrt{(\sigma_1 - \sigma_2)^2 a_1^2 a_2^2 + (\sigma_2 - \sigma_3)^2 a_2^2 a_3^2 + (\sigma_3 - \sigma_1)^2 a_3^2 a_1^2} \\
 &= \frac{\sqrt{I_1 I_2 I_3 - 9 I_3^2}}{I_2} = \tau_{SMP}
 \end{aligned}$$



$$\begin{aligned}
 X &= \frac{t_S}{t_N} = \frac{\tau_{SMP}}{\sigma_{SMP}} = \frac{2}{3} \sqrt{\frac{(\sigma_1 - \sigma_2)^2}{4\sigma_1\sigma_2} + \frac{(\sigma_2 - \sigma_3)^2}{4\sigma_2\sigma_3} + \frac{(\sigma_3 - \sigma_1)^2}{4\sigma_3\sigma_1}} \\
 &= \frac{2}{3} \sqrt{\tan^2 \phi_{mo12} + \tan^2 \phi_{mo23} + \tan^2 \phi_{mo13}} \\
 &= \sqrt{\frac{I_1 I_2}{9 I_3} - 1} = \text{const.}
 \end{aligned}$$

or

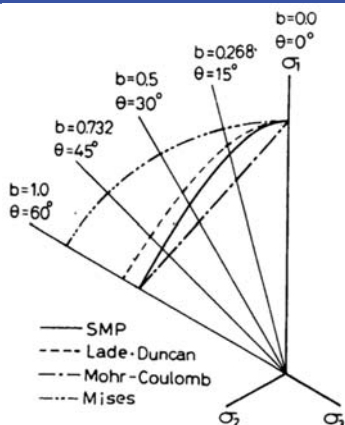
$$\frac{I_1 I_2}{I_3} = \text{const.}$$



— SMP ($X=0.2, 0.63$)
 - - - (Extended) Mises ($\eta=0.45, 1.36$)
 - · - · Mohr-Coulomb ($\phi=12.0^\circ, 33.8^\circ$)

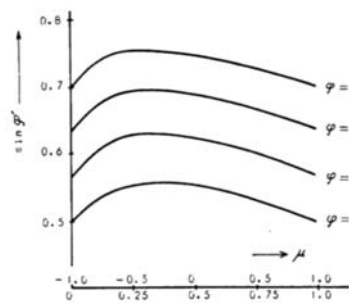


参考: 相異なる3主応力下の破壊規準(既報の実験と提案式)

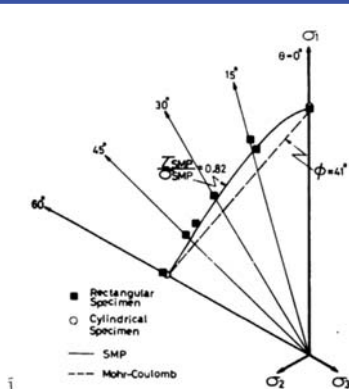


正八面体面上で表した4つの規準
 ここに、Lade-Duncan(1975)規準:

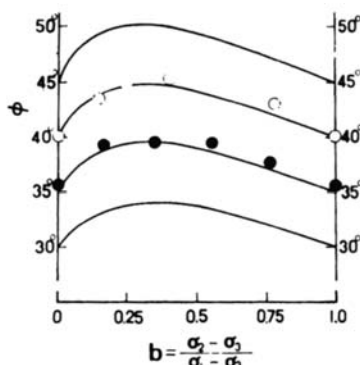
$$I_1^3 / I_3 = \text{const.}$$



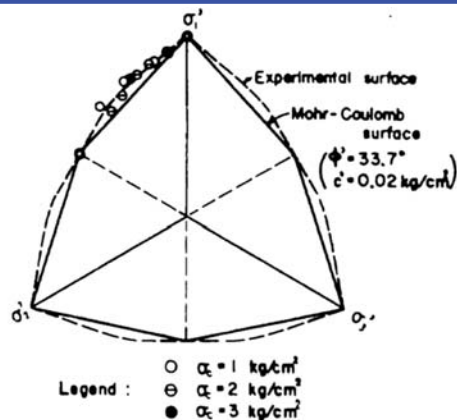
$\phi \sim b$ 関係で表した佐竹(1973)の規準:
 $\sin^2 \phi_{12} + \sin^2 \phi_{23} + \sin^2 \phi_{13} = \text{const.}$



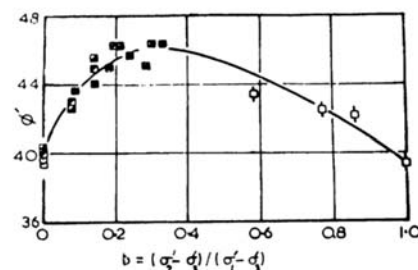
正八面体面上で表したSMP規準
 および砂の実測値.



$\phi \sim b$ 関係で表したSMP規準と
 Sutherland et. al. (1969)による砂の実測値



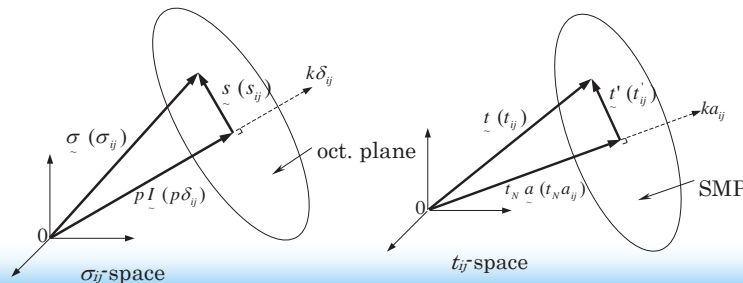
正八面体面上で表した正規圧密粘土の破壊時の実測値(柴田・軽部(1965))



$\phi \sim b$ 関係で表した砂の実測値
 (Ramamurthy・Rawatt(1973)) 18

通常および t_{ij} モデルで使われる応力・ひずみ増分パラメータ

	ordinary concept	t_{ij} concept
tensor normal to reference plane	δ_{ij} (unit tensor)	a_{ij} (tensor normal to SMP)
stress tensor	σ_{ij}	t_{ij}
mean stress	$p = \sigma_{ij} \delta_{ij} / 3$	$t_N = t_{ij} a_{ij}$
deviatoric stress tensor	$s_{ij} = \sigma_{ij} - p \delta_{ij}$	$t'_{ij} = t_{ij} - t_N a_{ij}$
deviatoric stress	$q = \sqrt{(3/2)s_{ij}s_{ij}}$	$t_S = \sqrt{t'_{ij}t'_{ij}}$
stress ratio tensor	$\eta_{ij} = s_{ij} / p$	$x_{ij} = t'_{ij} / t_N$
stress ratio	$\eta = q / p$	$X = t_S / t_N$
strain increment normal to reference plane	$d\varepsilon_v = d\varepsilon_{ij} \delta_{ij}$	$d\varepsilon_N^* = d\varepsilon_{ij} a_{ij}$
deviatoric strain increment tensor	$de_{ij} = d\varepsilon_{ij} - d\varepsilon_v \delta_{ij} / 3$	$d\varepsilon'_{ij} = d\varepsilon_{ij} - d\varepsilon_N^* a_{ij}$
strain increment parallel to reference plane	$d\varepsilon_d = \sqrt{(2/3)de_{ij}de_{ij}}$	$d\varepsilon_S^* = \sqrt{d\varepsilon'_{ij}d\varepsilon'_{ij}}$



19

降伏関数の微分計算(1/2)

- 降伏関数の応力項

$$F = \ln \frac{t_{N1}}{t_{N0}} = \ln \frac{t_N}{t_{N0}} + \zeta(X) \quad (\text{where } X = t_S / t_N) \quad (\text{a})$$

- 降伏関数 F の t_{ij} による微分

$$\frac{\partial F}{\partial t_{ij}} = \frac{\partial F}{\partial t_N} \frac{\partial t_N}{\partial t_{ij}} + \frac{\partial F}{\partial X} \frac{\partial X}{\partial t_{ij}} \quad (\text{b})$$

$$\frac{\partial F}{\partial t_N} = \frac{1}{t_N} \quad (\text{c})$$

$$\frac{\partial t_N}{\partial t_{ij}} = \frac{\partial (t_{kl} a_{kl})}{\partial t_{ij}} = a_{ij} \quad (\text{d})$$

$$\frac{\partial F}{\partial X} = \zeta'(X) \quad (\text{e})$$

$$\frac{\partial X}{\partial t_{ij}} = \frac{\partial (\sqrt{x_{mn} x_{mn}})}{\partial x_{kl}} \frac{\partial x_{kl}}{\partial t_{ij}} = \frac{1}{X \cdot t_N} (x_{ij} - X^2 a_{ij}) \quad (\text{f})$$

where

$$x_{ij} = \frac{t'_{ij}}{t_N} = \frac{t_{ij} - t_N a_{ij}}{t_N} = \frac{t_{ij}}{t_N} - a_{ij} \quad (\text{g})$$

$$\frac{\partial X}{\partial x_{kl}} = \frac{\partial (\sqrt{x_{mn} x_{mn}})}{\partial x_{kl}} = \frac{x_{kl}}{X} \quad (\text{h})$$

$$\frac{\partial x_{kl}}{\partial t_{ij}} = \frac{\partial}{\partial t_{ij}} \left(\frac{t_{kl}}{t_N} - a_{kl} \right) = \frac{1}{t_N} \{ \delta_{ik} \delta_{jl} - (x_{kl} + a_{kl}) a_{ij} \} \quad (\text{i})$$

20

降伏関数の微分計算(2/2)

- 降伏関数 F の σ_{ij} による微分

$$\frac{\partial F}{\partial \sigma_{ij}} = \frac{\partial F}{\partial t_N} \frac{\partial t_N}{\partial \sigma_{ij}} + \frac{\partial F}{\partial X} \frac{\partial X}{\partial \sigma_{ij}} \quad (j)$$

$$\frac{\partial t_N}{\partial \sigma_{ij}} = \frac{\partial}{\partial \sigma_{ij}} \left(3 \frac{I_3}{I_2} \right) = -3 \frac{I_3}{I_2^2} \frac{\partial I_2}{\partial \sigma_{ij}} + 3 \frac{1}{I_2} \frac{\partial I_3}{\partial \sigma_{ij}} \quad (k)$$

$$\frac{\partial X}{\partial \sigma_{ij}} = \frac{\partial}{\partial \sigma_{ij}} \left(\sqrt{\frac{I_1 I_2}{9 I_3} - 1} \right) = \frac{1}{2X} \left(\frac{I_2}{9 I_3} \frac{\partial I_1}{\partial \sigma_{ij}} + \frac{I_1}{9 I_3} \frac{\partial I_2}{\partial \sigma_{ij}} - \frac{I_1 I_2}{9 I_3^2} \frac{\partial I_3}{\partial \sigma_{ij}} \right) \quad (l)$$

where

$$\frac{\partial I_1}{\partial \sigma_{ij}} = \frac{\partial \sigma_{kk}}{\partial \sigma_{ij}} = \delta_{ij} \quad (m)$$

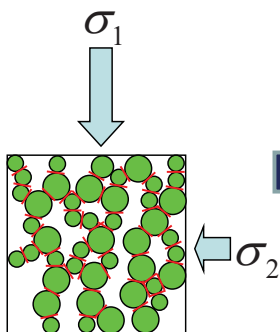
$$\frac{\partial I_2}{\partial \sigma_{ij}} = \frac{\partial}{\partial \sigma_{ij}} \left(\frac{(\sigma_{kk})^2 - \sigma_{lm} \sigma_{ml}}{2} \right) = \sigma_{kk} \delta_{ij} - \sigma_{ij} \quad (n)$$

$$\frac{\partial I_3}{\partial \sigma_{ij}} = \frac{\partial}{\partial \sigma_{ij}} \left(\frac{e_{klm} e_{opq} \sigma_{ko} \sigma_{lp} \sigma_{mq}}{6} \right) = \frac{1}{2} e_{ilm} e_{jpn} \sigma_{lp} \sigma_{mq} \quad (o)$$

21

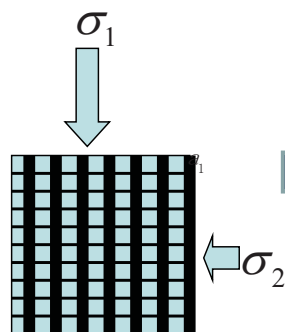
t_{ij} の概念の意味

土のような
粒状体

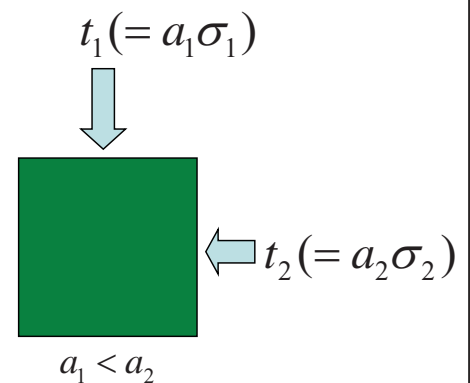


小田(1972)の
微視的考察

異方的な連続
体とみなす



等価な等方性
連続体とみる

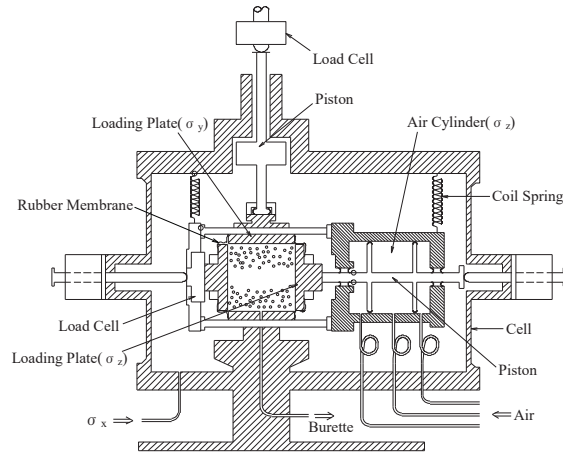
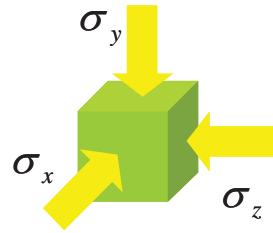
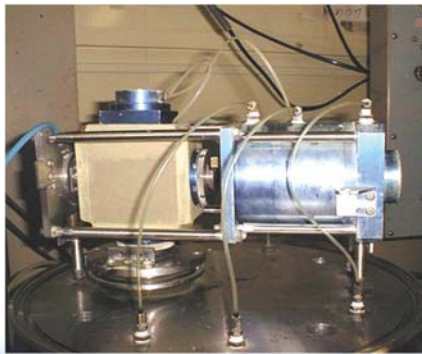


$$\therefore a_1 : a_2 = \frac{1}{\sqrt{\sigma_1}} : \frac{1}{\sqrt{\sigma_2}}$$

$$t_{ij} = a_{ik} \sigma_{kj}$$

22

3主応力制御試験機



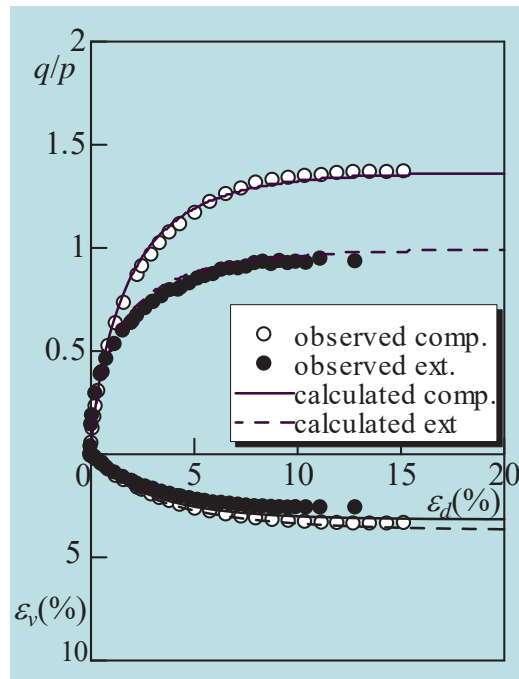
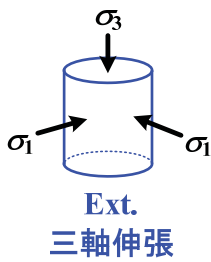
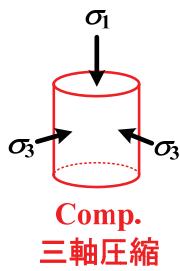
23

藤森粘土の材料パラメーター

λ	0.090	Same parameters as Cam clay model
κ	0.020	
$N(e_N \text{ at } p = 98\text{kPa})$	0.83	
$R_{CS} = (\sigma_1 / \sigma_3)_{CS(\text{comp.})}$	3.5	
v_e	0.2	
β	1.5	Shape of yield surface (same as original Cam clay if $\beta = 1$)

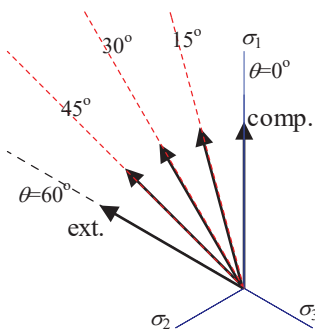
24

正規圧密粘土の三軸圧縮および伸張試験結果と解析結果

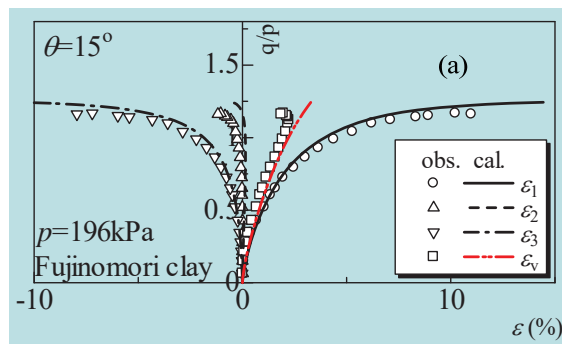


$p=196\text{kPa}$

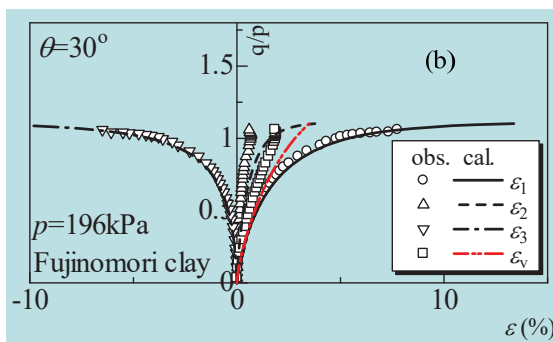
正規圧密粘土の3主応力制御試験



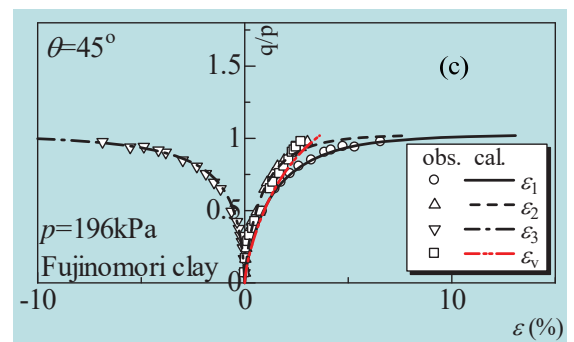
正 8 面体面状の応力経路



$\theta=15^\circ$

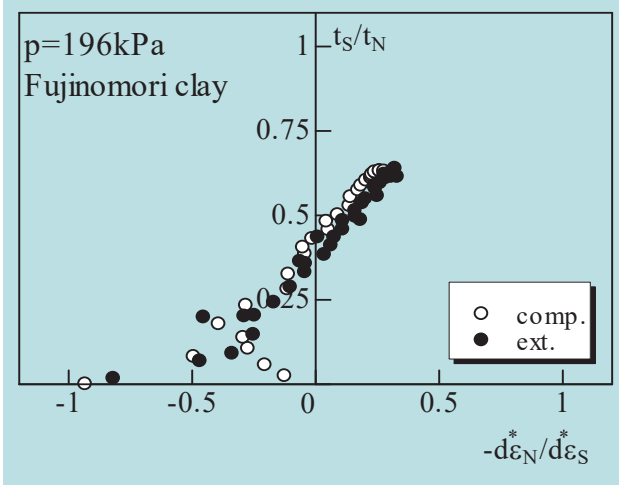


$\theta=30^\circ$

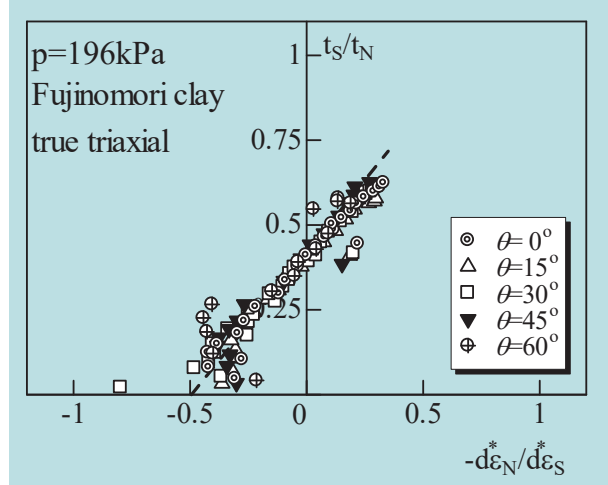


$\theta=45^\circ$

正規圧密粘土のstress-dilatancy関係の実測値



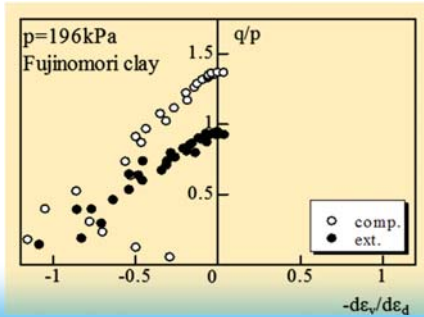
三軸圧縮および伸張試験



3主応力制御試験

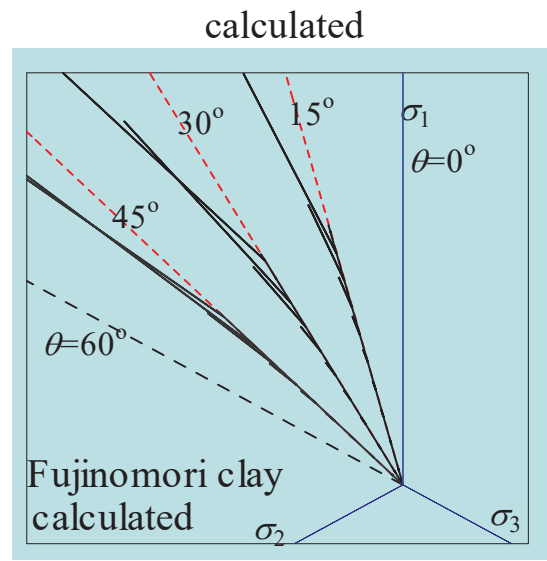
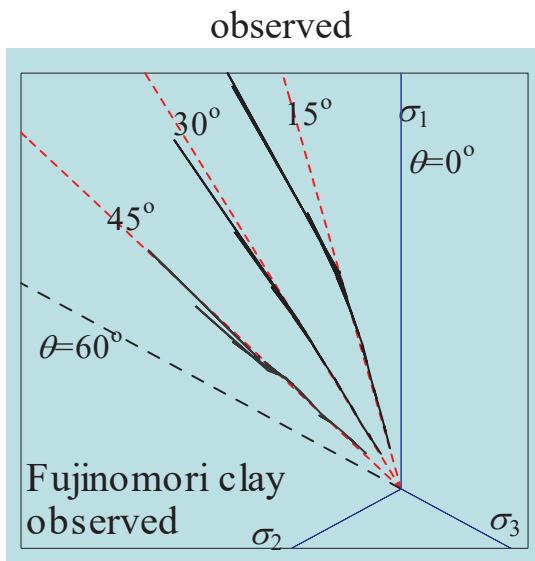
$$t_s/t_N : d\varepsilon_N^*/d\varepsilon_S^* \text{ 関係}$$

cf.

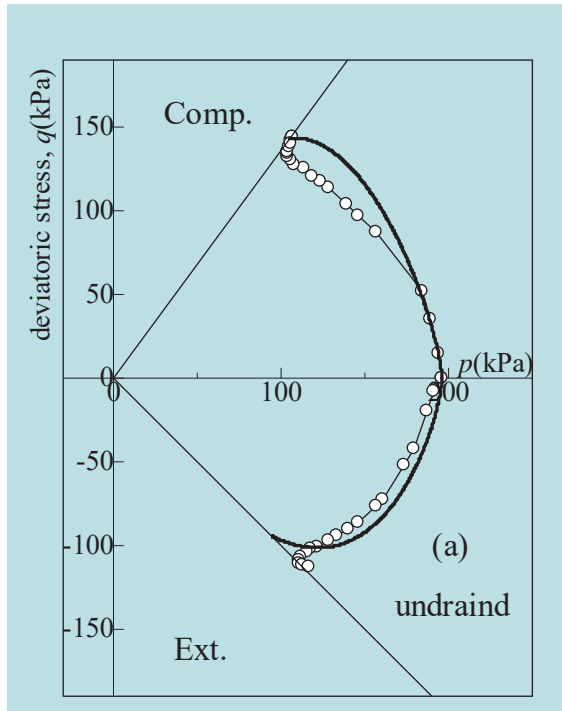


$$q/p \text{ vs. } d\varepsilon_v/d\varepsilon_d$$

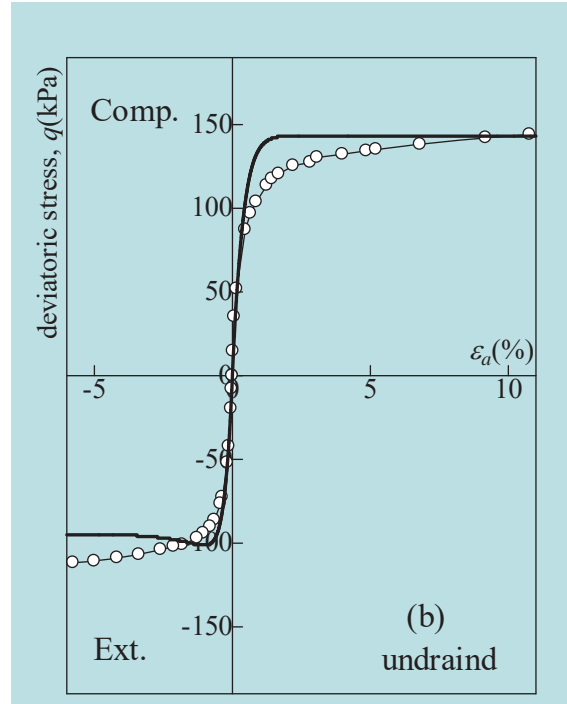
正規圧密粘土の3主応力制御試験



正8面体面上の応力経路とひずみ増分方向



有効応力経路



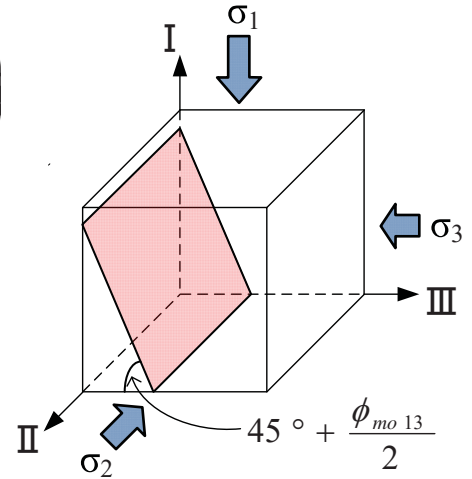
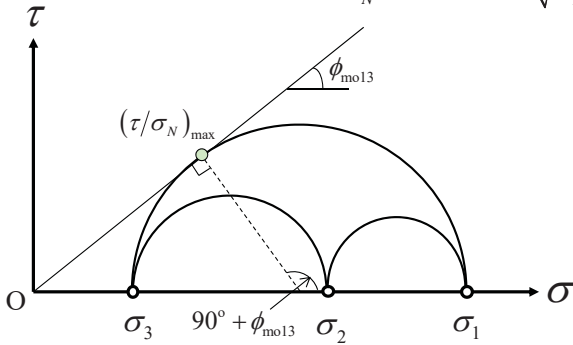
偏差応力～偏差ひずみ関係

滑動面から t_{ij} へ (付録)

付録：滑動面から t_{ij} へ—滑動面

村山(1964)

$$\frac{\tau}{\sigma_N} = \tan \phi_{mo} = \frac{\sigma_1 - \sigma_3}{2\sqrt{\sigma_1\sigma_3}} = \frac{1}{2} \left(\sqrt{\frac{\sigma_1}{\sigma_3}} - \sqrt{\frac{\sigma_3}{\sigma_1}} \right)$$



$$\left\{ \begin{aligned} \sigma_N &= \sigma_1 \cos^2 \left(45^\circ + \frac{\phi_{mo13}}{2} \right) + \sigma_3 \sin^2 \left(45^\circ + \frac{\phi_{mo13}}{2} \right) = \frac{2\sigma_1\sigma_3}{\sigma_1 + \sigma_3} \\ \tau &= (\sigma_1 - \sigma_3) \cos \left(45^\circ + \frac{\phi_{mo13}}{2} \right) \sin \left(45^\circ + \frac{\phi_{mo13}}{2} \right) = \frac{(\sigma_1 - \sigma_3)\sqrt{\sigma_1\sigma_3}}{\sigma_1 + \sigma_3} \end{aligned} \right.$$



$$\left\{ \begin{aligned} d\varepsilon_N &= d\varepsilon_1 \cos^2 \left(45^\circ + \frac{\phi_{mo13}}{2} \right) + d\varepsilon_3 \sin^2 \left(45^\circ + \frac{\phi_{mo13}}{2} \right) \\ \frac{d\gamma}{2} &= (d\varepsilon_1 - d\varepsilon_3) \cos \left(45^\circ + \frac{\phi_{mo13}}{2} \right) \sin \left(45^\circ + \frac{\phi_{mo13}}{2} \right) \end{aligned} \right.$$

□ τ/σ_N □ $d\varepsilon_N/d\gamma$ (stress-dilatancy式)

□ τ/σ_N □ γ 関係

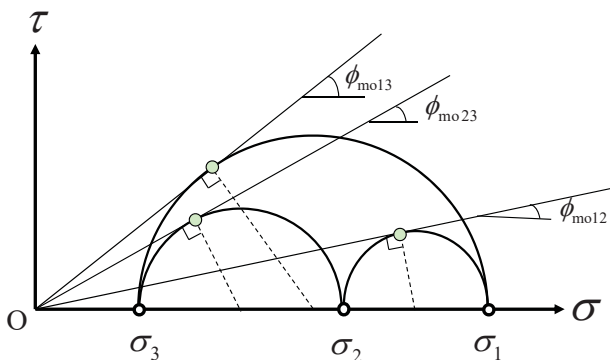


(応力とひずみ増分の共軸性)

$$\left\{ \begin{aligned} \varepsilon_1 &= f(\sigma_1/\sigma_3) \\ \varepsilon_3 &= g(\sigma_1/\sigma_3) \end{aligned} \right.$$

付録：滑動面から t_{ij} へ—複合滑動面

松岡(1974)



$$\left\{ \begin{aligned} \varepsilon_1 &= f(\sigma_1/\sigma_3) + f(\sigma_1/\sigma_2) \\ \varepsilon_2 &= f(\sigma_2/\sigma_3) + g(\sigma_1/\sigma_2) \\ \varepsilon_3 &= g(\sigma_1/\sigma_3) + g(\sigma_2/\sigma_3) \end{aligned} \right.$$

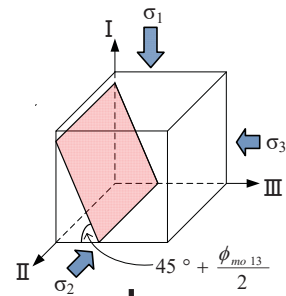
e.g.,

comp.: $\varepsilon_1 = 2f(\sigma_1/\sigma_3), \varepsilon_3 = g(\sigma_1/\sigma_3)$

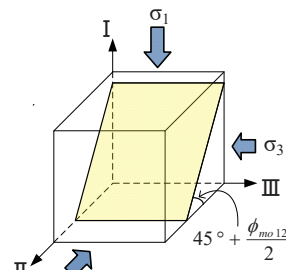
ext.: $\varepsilon_1 = f(\sigma_1/\sigma_3), \varepsilon_3 = 2g(\sigma_1/\sigma_3)$

plane st.: $\varepsilon_1 \square f(\sigma_1/\sigma_3), \varepsilon_3 \square g(\sigma_1/\sigma_3)$

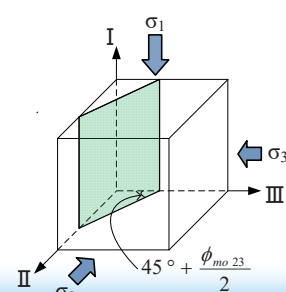
$$\left\{ \begin{aligned} d\varepsilon_1 &= d\varepsilon_{\text{major}(13)} + d\varepsilon_{\text{major}(12)} \\ d\varepsilon_2 &= d\varepsilon_{\text{major}(23)} + d\varepsilon_{\text{minor}(12)} \\ d\varepsilon_3 &= d\varepsilon_{\text{minor}(13)} + d\varepsilon_{\text{minor}(23)} \end{aligned} \right.$$



+

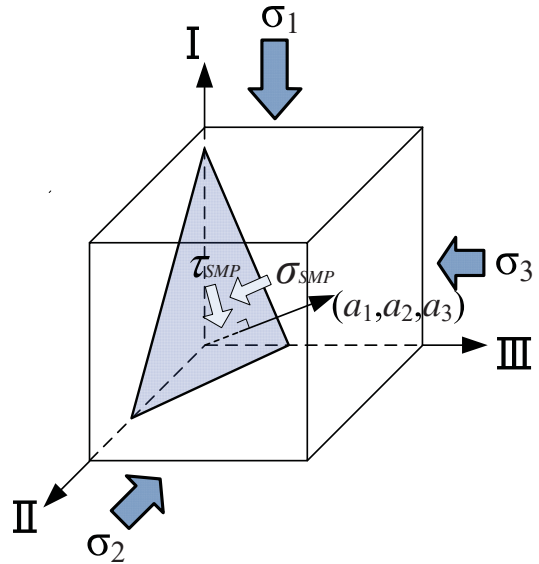
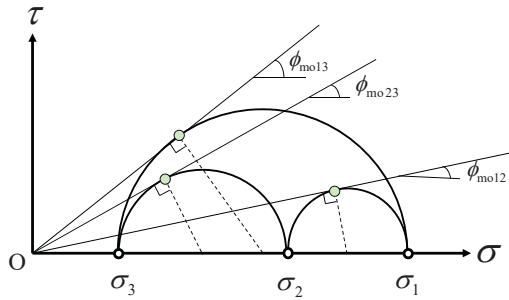


+



付録：滑動面から t_{ij} へ-空間滑動面(SMP)

松岡・中井(1974)



$$\begin{cases} \sigma_{SMP} = \sigma_1 a_1^2 + \sigma_2 a_2^2 + \sigma_3 a_3^2 \\ \tau_{SMP} = \sqrt{(\sigma_1 - \sigma_2)^2 a_1^2 a_2^2 + (\sigma_2 - \sigma_3)^2 a_2^2 a_3^2 + (\sigma_3 - \sigma_1)^2 a_3^2 a_1^2} \end{cases}$$

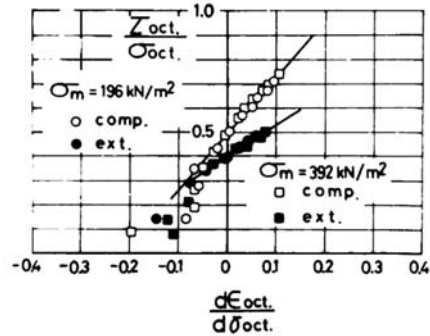
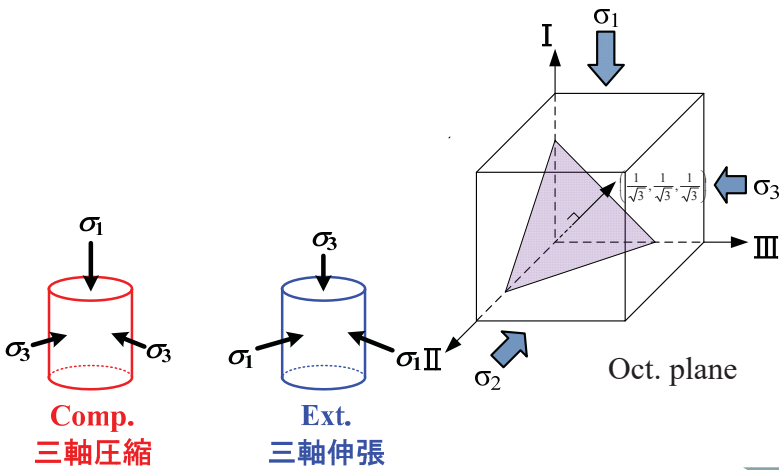
ここに、 (a_1, a_2, a_3) はSMPの方向余弦

$$a_1 = \sqrt{\frac{I_3}{I_2 \sigma_1}}, \quad a_2 = \sqrt{\frac{I_3}{I_2 \sigma_2}}, \quad a_3 = \sqrt{\frac{I_3}{I_2 \sigma_3}}$$

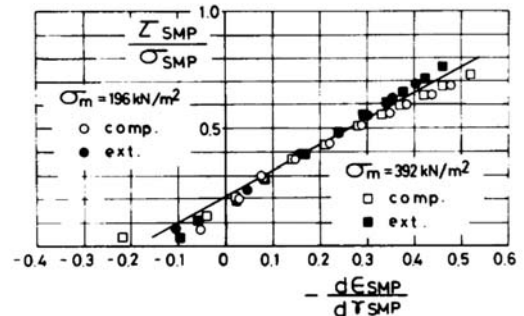
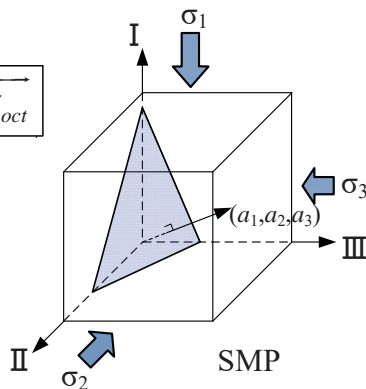
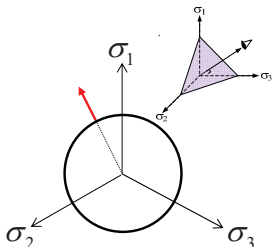
(I_2, I_3 : 応力の2次、3次の不変量)

$$\begin{cases} d\varepsilon_{SMP} = d\varepsilon_1 a_1^2 + d\varepsilon_2 a_2^2 + d\varepsilon_3 a_3^2 \\ \frac{d\gamma_{SMP}}{2} = \sqrt{(d\varepsilon_1 - d\varepsilon_2)^2 a_1^2 a_2^2 + (d\varepsilon_2 - d\varepsilon_3)^2 a_2^2 a_3^2 + (d\varepsilon_3 - d\varepsilon_1)^2 a_3^2 a_1^2} \end{cases}$$

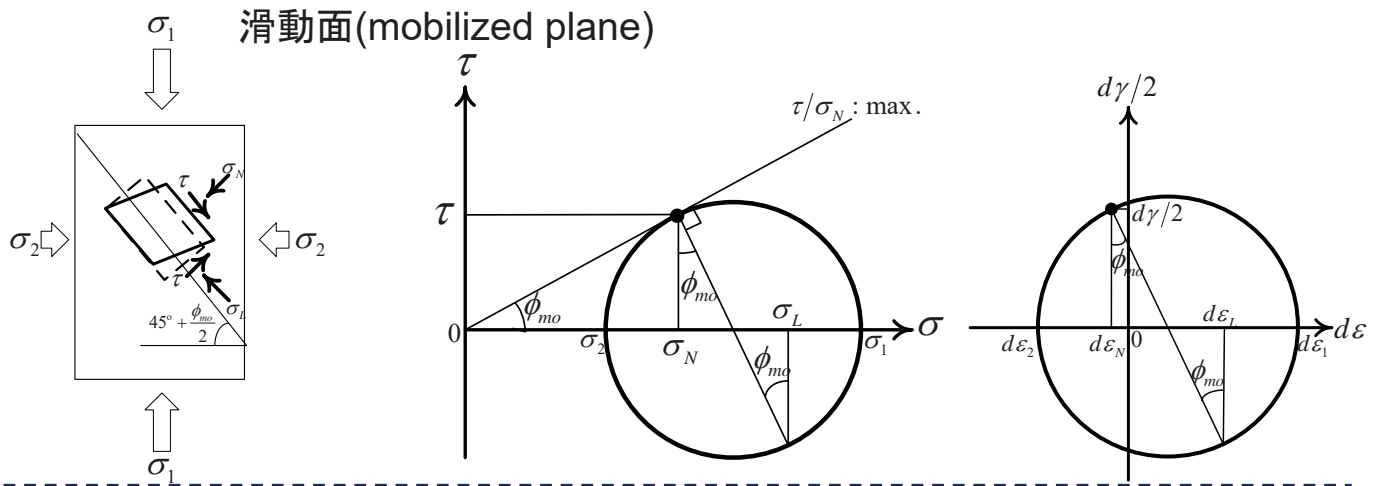
付録：滑動面から t_{ij} へ-空間滑動面(SMP)



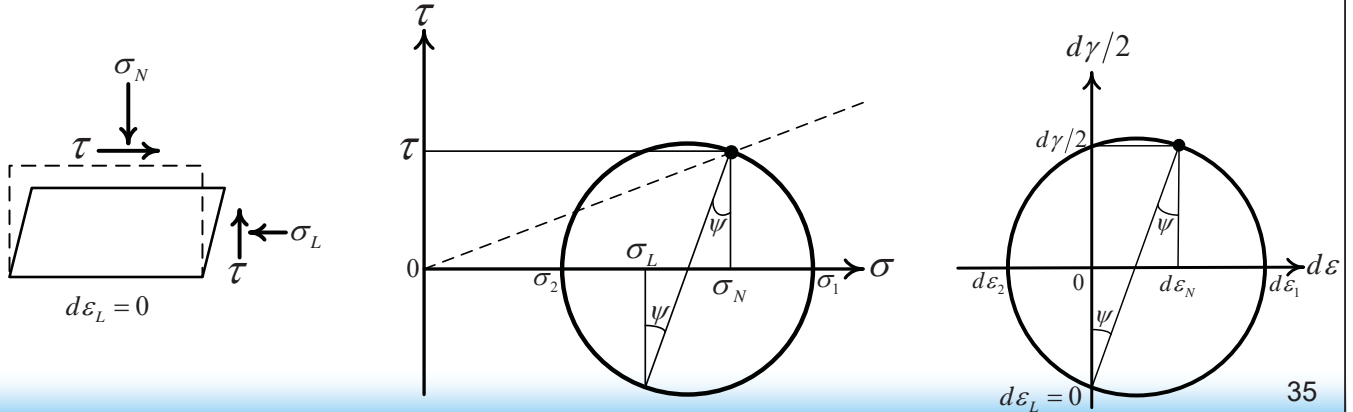
$$\vec{\tau}_{SMP} \parallel d\gamma_{SMP} = \vec{\tau}_{oct} \parallel d\gamma_{oct}$$



付録：滑動面から t_{ij} へ—滑動面での挙動と単純せん断挙動の違い

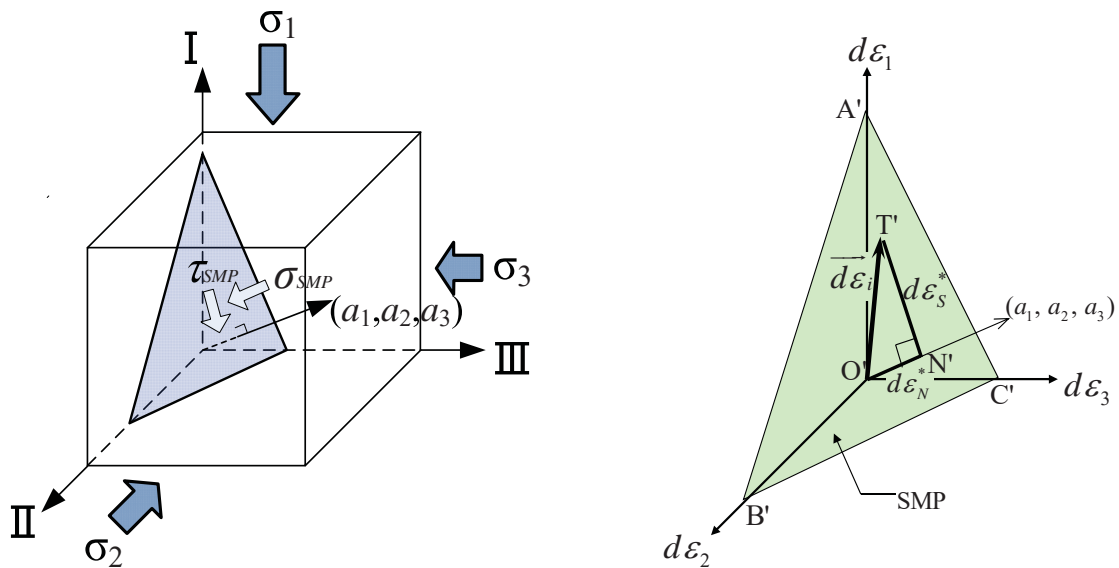


単純せん断(simple shear)



付録：滑動面から t_{ij} へ—拡張した空間滑動面(SMP*)

中井・松岡(1980, 1983)

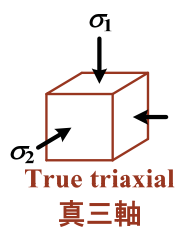
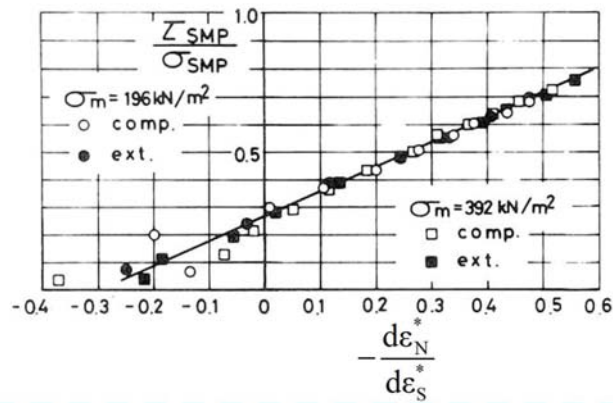
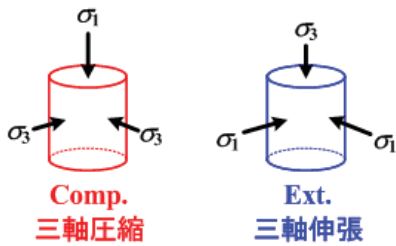


$$\begin{cases} \sigma_{SMP} = \sigma_1 a_1^2 + \sigma_2 a_2^2 + \sigma_3 a_3^2 \\ \tau_{SMP} = \sqrt{(\sigma_1 - \sigma_2)^2 a_1^2 a_2^2 + (\sigma_2 - \sigma_3)^2 a_2^2 a_3^2 + (\sigma_3 - \sigma_1)^2 a_3^2 a_1^2} \end{cases}$$

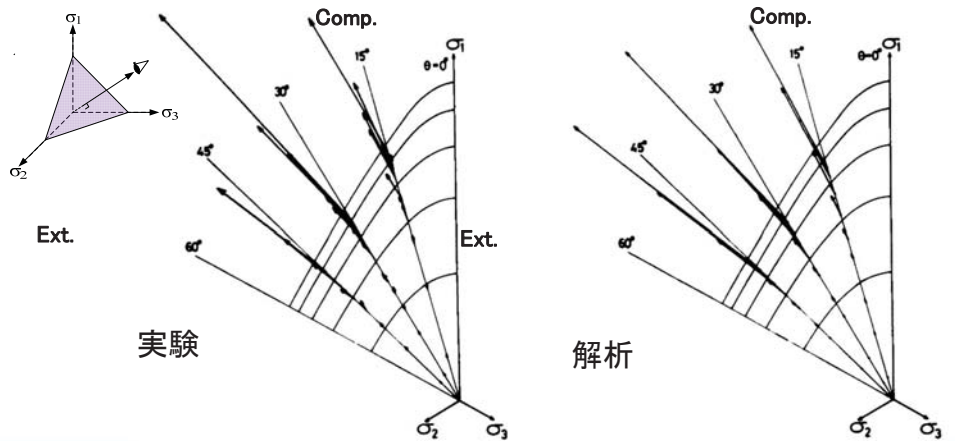


$$\begin{cases} d\epsilon_N^* = d\epsilon_1 a_1 + d\epsilon_2 a_2 + d\epsilon_3 a_3 \\ d\epsilon_s^* = \sqrt{(d\epsilon_1 a_2 - d\epsilon_2 a_1)^2 + (d\epsilon_2 a_3 - d\epsilon_3 a_2)^2 + (d\epsilon_3 a_1 - d\epsilon_1 a_3)^2} \end{cases}$$

付録：滑動面から t_{ij} へ拡張した空間滑動面(SMP*)



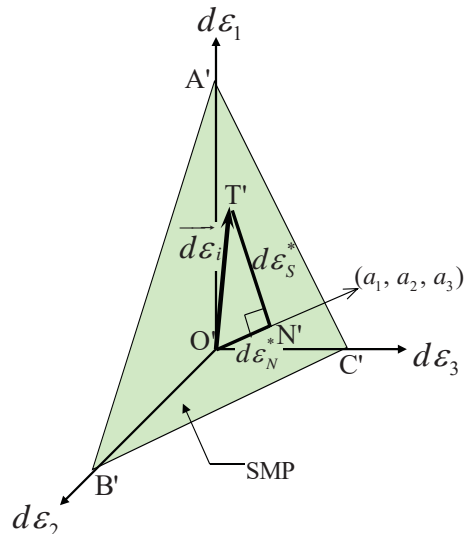
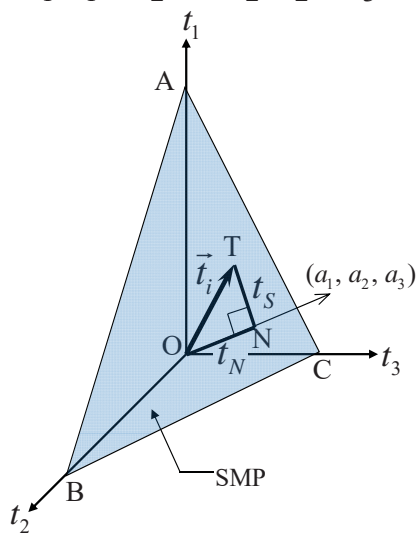
$$\vec{\tau}_{SMP} \parallel d\epsilon_S^*$$



付録：滑動面から t_{ij} へ修正応力 t_{ij} の概念

中井・三原(1984)

$t_1 = a_1 \sigma_1, t_2 = a_2 \sigma_2, t_3 = a_3 \sigma_3$ とすると



$$\begin{cases} t_N = \sigma_{SMP} = t_1 a_1 + t_2 a_2 + t_3 a_3 \\ t_S = \tau_{SMP} = \sqrt{(t_1 a_2 - t_2 a_1)^2 + (t_2 a_3 - t_3 a_2)^2 + (t_3 a_1 - t_1 a_3)^2} \end{cases}$$



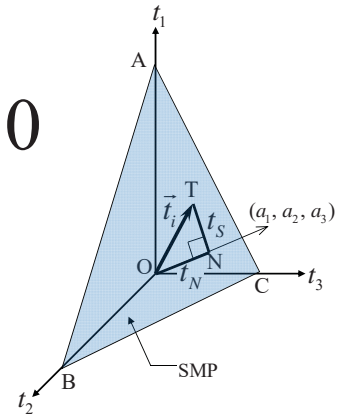
$$\begin{cases} d\epsilon_N^* = d\epsilon_1 a_1 + d\epsilon_2 a_2 + d\epsilon_3 a_3 \\ d\epsilon_S^* = \sqrt{(d\epsilon_1 a_2 - d\epsilon_2 a_1)^2 + (d\epsilon_2 a_3 - d\epsilon_3 a_2)^2 + (d\epsilon_3 a_1 - d\epsilon_1 a_3)^2} \end{cases}$$

弾塑性論への適用

$$f = F(t_N, X = t_S / t_N) - H = 0$$

$$d\varepsilon_{ij}^p = \Lambda \frac{\partial F}{\partial t_{ij}}$$

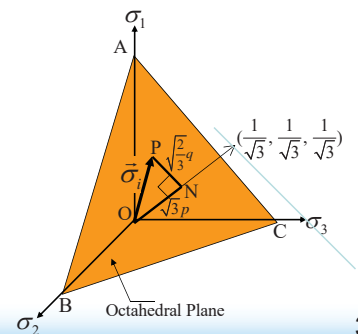
(正の係数 Λ は $df=0$ から決まる)



従来の弾塑性論 (e.g., Cam clay model)

$$f = F(p, \eta = q / p) - H = 0$$

$$d\varepsilon_{ij}^p = \Lambda \frac{\partial F}{\partial \sigma_{ij}}$$



39

t_{ij} の概念の意味

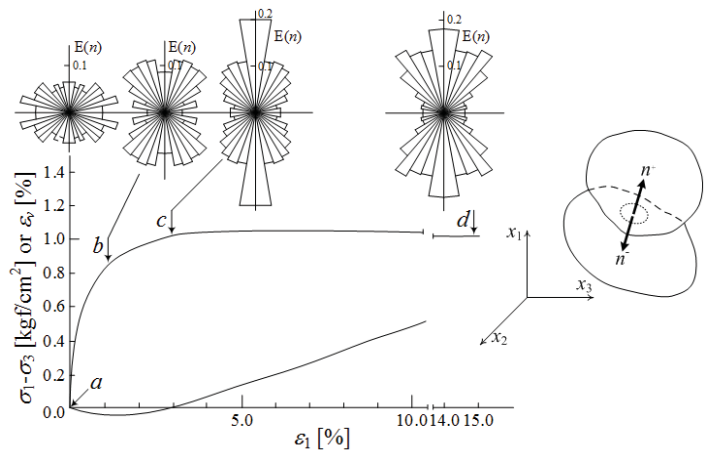
微視的観点から見た t_{ij} の意味

- Satake(1984)による構造テンソルの主値と応力比の関係

$$\varphi_1/\varphi_2 \approx (\sigma_1/\sigma_2)^{0.5}$$

- Satake(1982)による修正応力

$$\sigma_{ij}^* = (1/3)\varphi_{ik}^{-1}\sigma_{kj}$$

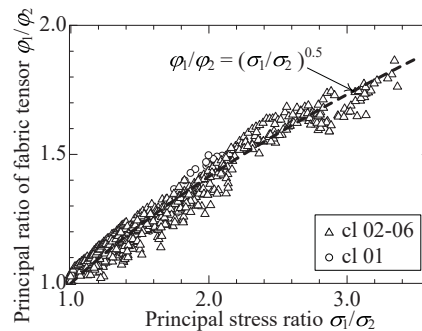


応力・ひずみ関係と粒子接平面の法線傾度分布(Oda, 1972; 1993)

- 修正応力 t_{ij} (Nakai & Mihara, 1984)

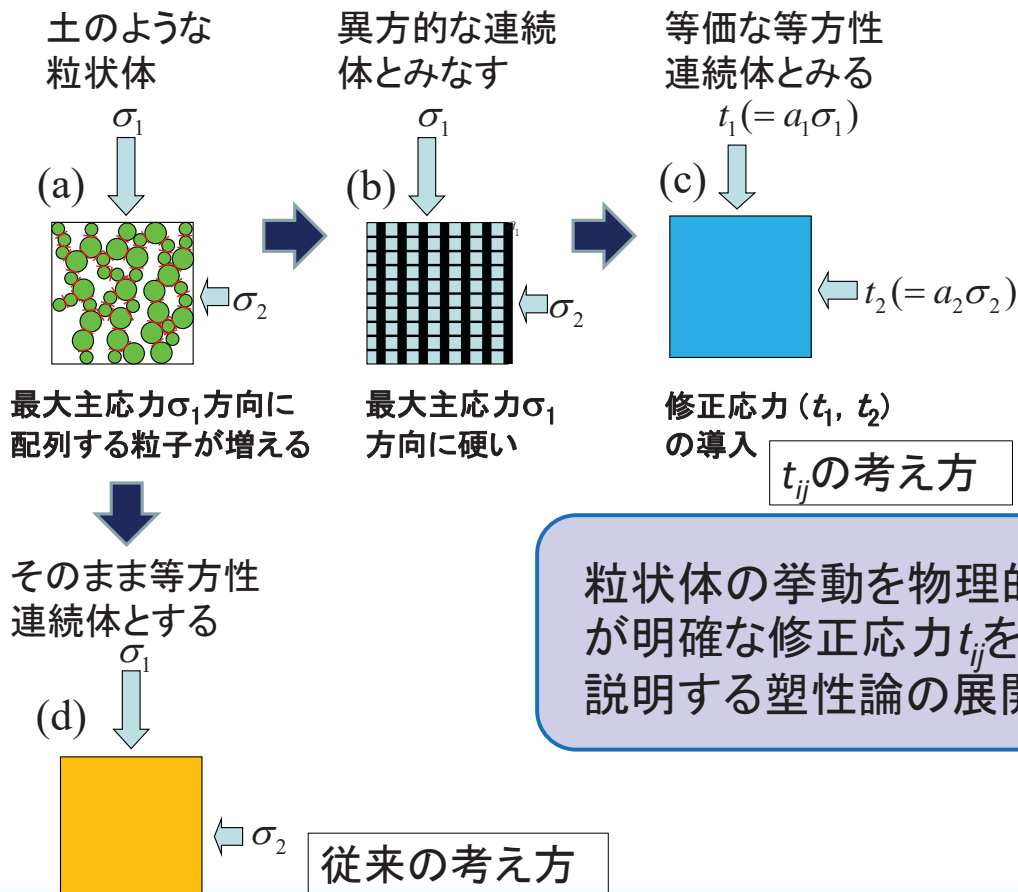
$$t_{ij} = a_{ik}\sigma_{kj}$$

$$(a_1, a_2, a_3) = \left(\sqrt{\frac{I_3}{I_2\sigma_1}}, \sqrt{\frac{I_3}{I_2\sigma_2}}, \sqrt{\frac{I_3}{I_2\sigma_3}} \right)$$

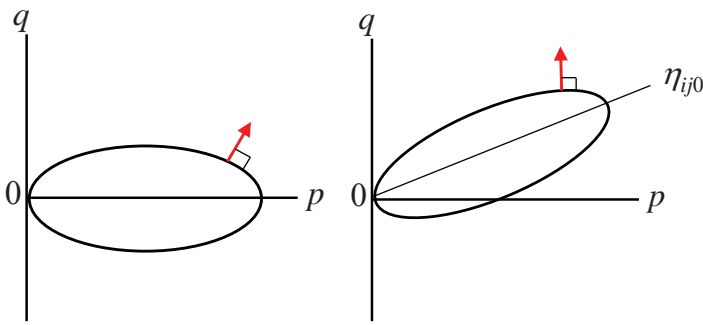


DEMIによる構造テンソルの主値の変化(Maeda et.al., 2006)

t_{ij} の概念に基づく塑性論と通常の塑性論



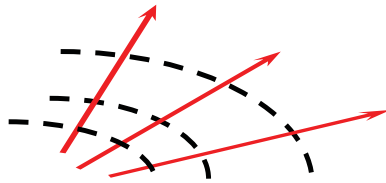
t_{ij} の概念における流れ則(直交則)の意味



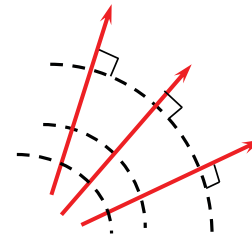
問題:
 等方硬化モデルはもちろんのこと、
 降伏関数を傾ける回転硬化モデル
 (左図)でも直行則を仮定して塑性
 ひずみ増分方向を決めている。

これは正しいか？

例えば、異方性地盤の透水問題では流れの方向は物理空間の等ポテンシャル面に直交せず、等方的な透水係数になるように変換した空間で直行則が成り立つ(ポテンシャル論)。



物理空間



変換空間

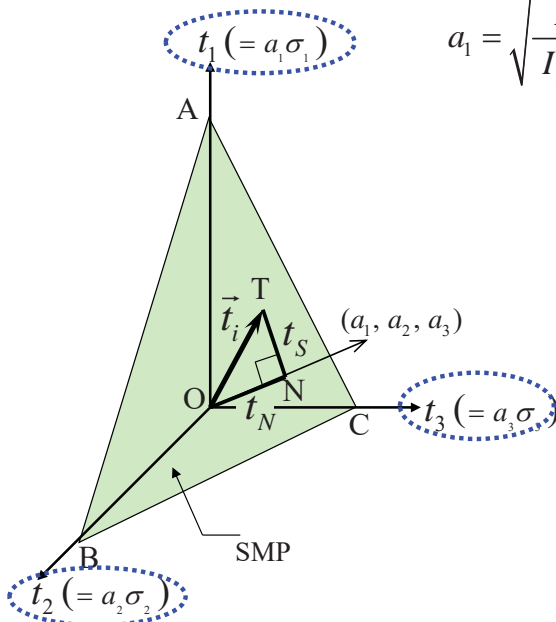
t_{ij} の考え方は応力(応力比)誘導異方性を有する材料を、等価な等方性を有する材料に置き換えることを意味する。そのような意味において、 σ_{ij} 空間ではなく t_{ij} 空間で直行則を考えるのが妥当だと言える。

43

t_{ij} の応力パラメータ(t_S , t_N)の意味

SMPの方向余弦：

$$a_1 = \sqrt{\frac{I_3}{I_2\sigma_1}}, \quad a_2 = \sqrt{\frac{I_3}{I_2\sigma_2}}, \quad a_3 = \sqrt{\frac{I_3}{I_2\sigma_3}}$$



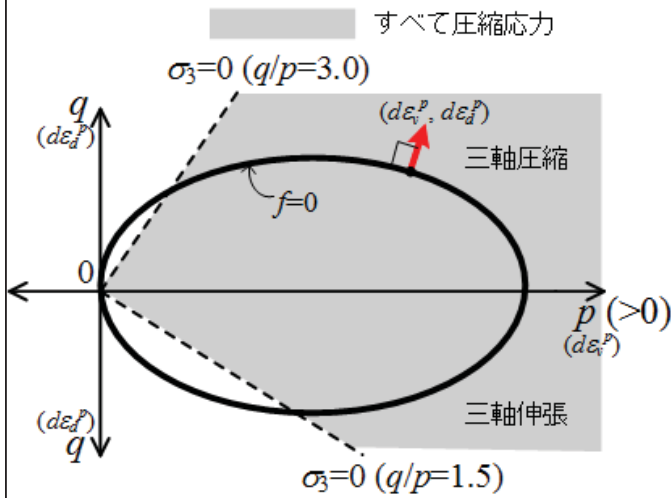
- ($t_1=a_1\sigma_1$, $t_2=a_2\sigma_2$, $t_3=a_3\sigma_3$)は応力誘導異方性を示す粒状性材料を等方的な連続体として扱うための修正応力である。

- 修正応力空間でSMPに垂直な成分 t_N と平行な成分 t_S を応力パラメータとし、応力比 $X = t_S / t_N$ でせん断挙動が説明できることは、粒状性材料は摩擦則でその挙動が支配されることを意味する。

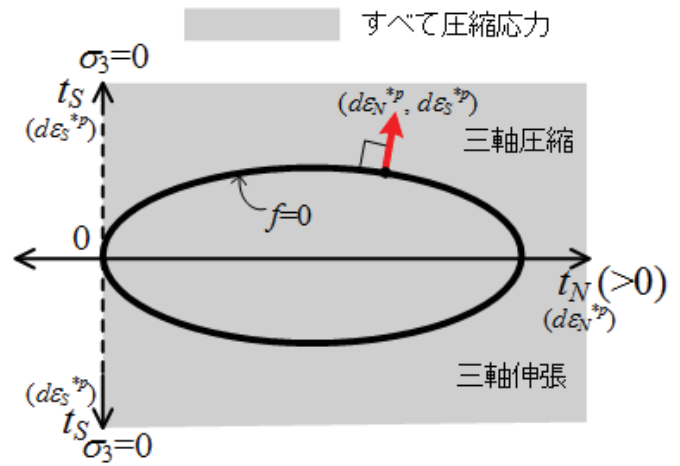
応力パラメータ：

$$\begin{cases} t_N = t_1 a_1 + t_2 a_2 + t_3 a_3 \\ t_S = \sqrt{(t_1 a_2 - t_2 a_1)^2 + (t_2 a_3 - t_3 a_2)^2 + (t_3 a_1 - t_1 a_3)^2} \end{cases}$$

44



(q, p)を応力パラメータとする弾塑性論の降伏曲面と流れ則



t_{ij} の概念に基づく (t_N, t_S) を応力パラメータとする弾塑性論の降伏曲面と流れ則

参考文献(1)

- 1) Oda M. (1972): The mechanism of fabric changes during compressional deformation of sand, *Soils and Foundations*, **12**(2), 1-18.
- 2) Oda M. (1993): Inherent and induced anisotropy in plasticity theory of granular soils, *Mechanics of Materials*, **16**, (1-2), 35-45.
- 3) Lade P.V. and Duncan J.M. (1975): Elastoplastic stress-strain theory for cohesionless soil, *Proc. of ASCE*, **101**(GT10), 1037-1053.
- 4) Maeda K. Hirabayashi H. and Ohmura A. (2006): Micromechanical influence of grain properties on deformation – failure behaviors of granular media by DEM. *Proc. of Geomechanics and Geotechnics of Particulate Media*, Yamaguchi, 173-179.
- 5) Matsuoka H. (1974): Stress-strain relationship of sand based on the mobilized plane, *Soils and Foundations*, **14**(2), 47-61.
- 6) Matsuoka H. and Nakai T. (1974): Stress-deformation and strength characteristics of soil under three different principal stresses, *Proc. of JSCE*, **232**, 59-70.
- 7) Murayama S. (1964): A theoretical consideration on a behavior of sand, *Proc. of IUTAM Symposium on Rheology and Soil Mechanics*, Grenoble, 146-159.
- 8) 中井照夫・松岡元 (1980): 3主応力下の土のせん断挙動に関する統一的解釈, *土木学会論文報告集*, **301**, 65-77.
- 9) Nakai T. and Matsuoka H. (1983): Shear behavior of sand and clay under three-dimensional stress condition, *Soils and Foundations*, **23**(2), 26-42.
- 10) Nakai T. and Mihara Y. (1984): A new mechanical quantity for soils and its application to elastoplastic constitutive models, *Soils and Foundations*, **24**(2), 82-94.
- 11) Nakai T. and Matsuoka H. (1986) A generalized elastoplastic constitutive model for clay in three-dimensional stresses, *Soils and Foundations*, **26**(3), 81-98.

参考文献(2)

- 12) Nakai T., Matsuoka H., Okuno M., Tsuzuki K. (1986): True triaxial tests on normally consolidated clay and analysis of the observed shear behavior using elastoplastic constitutive models, *Soils and Foundations*, **26**(4), 67-78.
- 13) 中井照夫 (2011): 地盤材料の構成モデル最前線 – 2.弾塑性論の解説とカムクレイモデルの適用性, *地盤工学会誌, 講座*, 59(4), 47-55.
- 14) 中井照夫 (2011): 地盤材料の構成モデル最前線 – 7.3主応力条件下での材料特性のモデル化, *地盤工学会誌, 講座*, 59(9), 66-75.
- 15) Nakai T. (2012): *Constitutive Modeling of Geomaterials: Principles and Applications*, CRC Press.
- 16) Nakai T., Shahin H.M., Hinokio M. and Kyokawa H. (2014): Why do constitutive models using (p and q) have problems?, *Proc of Int. Symp. On Geomechanics from Micro to Macro (IS-Cambridge 2014)*, 683-688.
- 17) Nakai T. (2018): Significance and usefulness of the t_{ij} concept, China-Euro Conference of Geotechnical Engineering, Vienna, *Desiderata Geotechnica*, Springer, 125-131
- 18) 中井照夫 (2018): 巨視的および微視的観点から見た t_{ij} の概念の意義, *地盤工学会誌*, 66(7), 18-21.
- 19) Ramamurthy T. and Rawat (1973): Shear strength of sand under general stress system, *Proc. of 8th ICSMFE*, 1.2, 339-342.
- 20) 佐武正雄(1973): 粒状態の降伏条件に関する1提案, 第28回土木学会年次学術講演会, 第3部, 91-92.
- 21) Satake M. (1982): Fabric tensor in granular materials. *Proc. of IUTAM Conference on Deformation and Failure of Granular Materials*, Delft, 63-68.
- 22) 佐武正雄 (1984): 地盤と土の異方性, *土と基礎*, 32(11), 5-12.
- 23) Shibata T. and Karube D. (1965): Influence of the variation of the intermediate principal stress on the mechanical properties of normally consolidated clay, *Proc. of 6th INCSMFE*, **1**, 359-363.
- 24) Surtherland H.B. and Mesdary M.S. (1969): The influence of the intermediate principal stress on the strength of sand, *Proc. of 7th ICSMFE*, **1**, 391-399.